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EVALUATING UNCERTAINTY WITH CONFIDENCE: A STUDY OF THE REPRESENTATIVE SCENARIO METHOD

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Abstract: The U.S. insurance industry is undergoing a fundamental shift from traditional formula-based reserving to principle-based reserving (PBR), driven by the increasing complexity of insurance products and heightened exposure to market risks such as interest rate and equity volatility. In response, working groups within the American Academy of Actuaries are developing and refining methodologies that better reflect the economic reality of modern insurance contracts.

Traditional reserving methods have relied on static, conservative assumptions within closed-form valuation models to ensure solvency under adverse conditions. However, the growing sophistication of insurance products—often embedded with derivative-like features—necessitates more dynamic and realistic approaches. PBR incorporates financial engineering techniques and focuses on the economic value of liabilities, with the Monte Carlo simulation emerging as the preferred tool for capturing complex, path-dependent behaviors.

Despite its accuracy, the Monte Carlo method is computationally intensive, particularly given the scale of in-force policy blocks, the complexity of model assumptions, and the extensive parameter space. As a result, improving computational efficiency has become a critical area of research. This paper explores scenario selection—a targeted approach to reducing computational burden without compromising the integrity of valuation outputs. By carefully selecting representative scenarios that preserve statistical properties and risk exposures, actuaries can maintain valuation precision while optimizing resource use.

This study highlights scenario selection as a promising technique in supporting the actuarial profession's transition to PBR, underscoring its practical significance in meeting regulatory expectations and adapting to a rapidly evolving financial landscape.

Keywords: Insurance industry, principle-based reserving, Monte Carlo method, computational efficiency, scenario selection.

1. Introduction

The insurance industry in United States has been going through a profound change. The reserve methodology has been shifting from a formula based reserving to the principle based reserving (PBR). The working groups of

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American Academy of Actuaries are developing the methodology and get adapted by the regulators. One driver / motivation of the change is the increasing complexity of products and increased market risk, equity risk additional to the interest rate risk, in those products.

The old fashion formula based reserve methods are mostly discounted cash flow models with conservative assumptions to account for adverse deviations. The PBR, on the other hand, usually are based on the economic value of the product and model the insurance policies with financial engineering concepts. As the insurance product becomes more complex, the derivatives embedded in the product would not be valued with the "closed form formula". The method of choice for valuation is the "Monte Carlo" method.

The Monte Carlo method usually requires significant computation power due to size of the in-force, complexity of the product, number of parameters. The actuaries have been trying to improve the computational efficiency of those methods. In this paper, we will discuss one of those efforts in scenario selection.

There are some proposals for smaller insurance companies to use the Representative Scenario Method (RSM), which requires less computational resource, but reflects the idea of PBR. The RSM calculates the reserve based on the liability projection of limited number of scenarios (for example, 5 scenarios). Please refer to [1], [2], [3] for concepts and applications of RSM. The choice of the scenario is critical to the resulting reserve. In this work, we modify the concept of the importance sampling to propose a methodology of choosing these limited scenarios. We will compare the RSM reserve based on our approach to the full Monte Carlo result.

2 Representative Scenario Method

X X X

X $E(R(X))$

There are a few versions of Representative Scenario Method (RSM). We will focus on the percentile method. Conceptually, for a function $R()$ of random variables , we will pick 5 representative scenarios at different percentile point, P_i, i = 1,2,3,4,5, in the distribution of , and use the probability of "landing" near the percentile point P_i, i = 1,2,3,4,5 as the weight for $R(P_i)$, the sum would be the estimate of the . In the context of insurance application, $R(X)$ would be the reserve function as the function of the key risk driver, the interest rate level. We formulate the approach below.

Let P_i, i = 1, 2, 3, 4, 5 be the percentile points. $0 < P_1 < P_2 < P_3 < P_4 < P_5 < 1$.

Let $KRD(X)$ is the cumulative distribution function of the random variable X of the key risk driver of the reserve.

$$X_i = KRD^{-1}(P_i)\ i = 1,2,3,4,5$$

Let $R(X)$= Reserve(X) be the reserve function of the key risk driver. The RSM estimates the reserve as

$R(X_1) * KRD(0.5 * (X_1 + X_2)) +$

$R(X_2) * \left(KRD(0.5 * (X_2 + X_3)\right.$

$R(X_3) * \left(KRD(0.5 * (X_2 + X_3)\right.$

$R(X_4) * \left(KRD(0.5 * (X_2 + X_3)\right.$

$R(X_5) * (1 - KRD(0.5 * (X_1 +$

$- KR\,D(0.5 * (X_1 + X_2))\Big) +$

$- KR\,D(0.5 * (X_1 + X_2))\Big)$

$- KR\,D(0.5 * (X_1 + X_2))\Big)$

$_2)))$

$+$

$+$

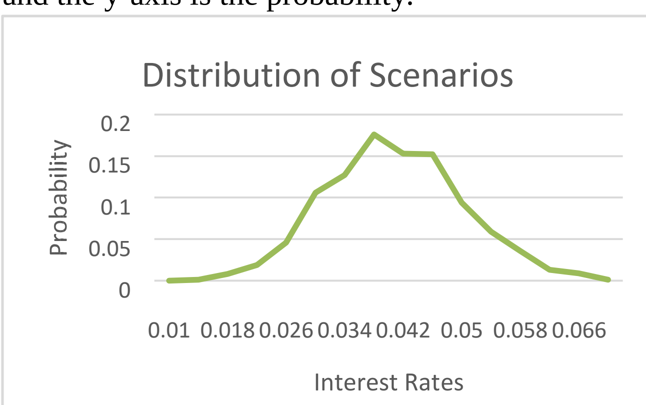
X

We measure the accuracy, appropriateness of the methodology used by comparison of the results of the RSM with the full simulation result.

We will illustrate the performance of RSM and our proposed methodology using the portfolio of a fixed income instrument and the derivative instrument. The fixed income instrument has a bullet payment at the maturity M. If the yield to maturity is r, the PV of the position is (1+r)^(-M). The derivative instrument is a swaption type of over the counter derivative with a payment notional (N) * Max (K-r, 0), where r is the market level, K is the strike rate. The value of the portfolio then would be P(r, M, K, N) = (1+r) ^ (-M) + N* Max (K-r, 0).

For the purpose of our exercise, we will choose a notional amount (N) so that the portfolio behaves as a mix of two types of positions, a fixed income position and a derivative position. If N is too large, then the option component dominates. If N is too small, then the fixed income component dominates.

We simulate the interest rate level r, with 1000 trials. The mean is 3.88% and the stdev at 0.92%. The following histogram provides a bit more information of the distribution of the 1000 scenarios. The x-axis is the rate level, and the y-axis is the probability.



The comparison of the full run and the RSM is listed in Table 1 for various portfolios, bond only, swaption only and well mixed portfolio of bond and swaption. The full simulation result is the average of the payoff of the 1000 scenarios. RSM result is obtained using $P_1 = 0.01, P_2 = 0.15, P_3 = 0.50,\ P_4 = 0.85\ and\ P_5 = 0.99$.

We observe that for the bond position with a $1 bullet payment, the RSM works very well. The RSM PV result of 0.332 is very close to the PV result of 0.331, with only 0.25% estimation error. The estimation error for the swaption varies greatly depending on the money-ness of the swaption. We denote the strike is at the money (ATM) if the strike equals the mean of the distribution, K=3.88%. The swaption 0.8 is the swaption where the strike is 0.8*3.88%. The swaption is out of money (OTM). The swaption 1.2 has a strike at 1.2*3.88%. This is a swaption in the money (ITM). We observe that the RSM works well for the ITM swaption with an error of 0.03%, but the error for the OTM swaption is relatively large at 12.52%. The portfolio, with a K= 0.8*3.88%, N=100, M=30 , has an error term at 3.01%.

Table 1 Performance of **RSM**

Portfolio	Full Simulation	RSM	Error%
Bond	0.331	0.332	0.25%
Swaption 0.8	0.096	0.108	12.52%
Swaption ATM	0.367	0.342	-6.83 %
Swaption 1.2	0.884	0.884	-0.03 %
Portfolio (Bond & 0.8 Swaption)	0.427	0.440	3.01%

As Table 1 shows, the RSM worked well on the bond like risk profile, but it has a much bigger error for option like payoffs. With the products in the market often with embedded option, the methodology proposed here would be beneficial. The purpose of this research is to propose a new method, which will reduce the error term for the portfolio.

3 Adjusted RSM

In this work, we propose a new methodologies, adjusted representative scenario method (ARSM), which combines the concept of the RSM with the concept of the importance sampling. The new methodology will pick the five percentile points, based on the payoff function and distribution. The methodology improves the accuracy of the RSM.

$E(g(X))$ $g(X)$
$|g(X)|$ X $g(X)$
$(g(X))$ $C * |g(X)|$ $h(X)$

The basic concept of the importance sampling in the Monte Carlo simulation is to pick a different and better distribution to simulate. If the goal is to use the Monte Carlo method to estimate where is a function of random variable . The best simulated distribution should be proportional to the with a distribution function of , where C is the scalar. The dilemma is that the integral is what we are computing in the first place. It is likely that we do not have the constant scalar . To avoid this dilemma, one option is to choose another function close to / similar to , say , the integral is available. $g(X)$ $h(X)$ $E(h(X))$

In our case, where is a reserve function of an insurance policy or a payoff function of a derivative. We would choose a function as a proxy function, which is as close to the actual payoff function as possible, then "pre-study and graph" the distribution of $h(X)$.

The concept is to study a proxy portfolio„s distribution, then use distribution to adjust RSM so that when the RSM is applied to the slightly different portfolio, it would perform better with a smaller error term.

This concept of the proxy portfolio and the sensitivity portfolio could be extended to similar situations in practice.

For example, a proxy portfolio would be a "standard" typical VA contract, and the sensitivity portfolio would be specific VA product of the company. In the case where a detailed analysis of the company specific product is not feasible, we would use the well-studied distribution of the well-studied "generic" product. With this approach, the adjusted RSM should provide a better estimation.

In this section, we will refer the portfolio we study in detail, with known payoff distribution, as the proxy portfolio, and the "unknown portfolio" as the sensitivity portfolio. We would formulate the adjusted RSM as follows. Let $RSP(X)$ = Reserve(X) be the reserve function of the key risk driver of the sensitivity portfolio over the reserve function of the proxy portfolio.

Let $PPP(X)$ be the payoff function of the proxy portfolio. Let $CumPPP(X)$ be the cumulative distribution function of the payoff, where X is the random variable of the key risk driver. The variable X has the cumulative distribution function $KRD(X)$.

Let P_i, i = 1, 2, 3, 4, 5 be the five percentile points for the RSM.

The adjusted RSM applies the RSM to the random variable which has the cumulative distribution function $CumPPP(X)$, and the RSP(X) as the payoff function. We have

$$X_i = CumPPP^{-1}(P_i) \; i = 1, 2, 3, 4, 5$$

And the ARSM estimates the reserves to be

$RSP(X_1) * CumPPP(0.5 * (X_1 + X_2))$ +
$RSP(X_2) * \left(CumPPP(0.5 * (X_2 + X_3)\right.$
$RSP(X_3) * \left(CumPPP(0.5 * (X_2 + X_3))\right)$
$RSP(X_4) * \left(CumPPP(0.5 * (X_2 + X_3))\right)$
$RSP(X_5) * (1 - CumPPP(0.5 * (X_1 +$
$- Cu\ mPPP(0.5 * (X_1 + X_2)))$+
$- Cum\ PPP(0.5 * (X_1 + X_2)))$
$- Cum\ PPP(0.5 * (X_1 + X_2)))$
)))
+
+
X_2

4 Results and Sensitivities

Table 2 below has the comparison of the result. The proxy portfolio has the swaption strike at 0.8*3.88% and bond maturity at 30. Note that 3.88% is the mean of the 1000 scenario. We consider the sensitivity of both to the bond maturity and to the swaption strike.

Table 2, Performance of Adjusted RSM (M = 30, Strike = 0.8)

Case #	duration sensitivity	strike sensitivity	proxy portfolio - RSM	sensitivity portfolio - RSM	sensitivity portfolio - adjusted RSM	Is Adjusted RSM better?
1	0	0.97	3.01%	0.62%	-2.09%	
2	0	0.98	3.01%	1.49%	-1.33%	TRUE
3	0	0.99	3.01%	2.30%	-0.63%	TRUE
4	0	1	3.01%	3.01%	0.00%	TRUE
5	0	1.01	3.01%	3.60%	0.52%	TRUE
6	0	1.02	3.01%	4.11%	0.96%	TRUE
7	-3	1	3.01%	2.74%	-0.16%	TRUE
8	-2	1	3.01%	2.83%	-0.11%	TRUE
9	-1	1	3.01%	2.92%	-0.06%	TRUE
10	0	1	3.01%	3.01%	0.00%	TRUE
11	1	1	3.01%	3.10%	0.06%	TRUE
12	2	1	3.01%	3.19%	0.12%	TRUE

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In Table 2, case # 1, 2, 3, 4, 5, 6 are results relative the swaption strike. Case #1 has the swaption strike of 0.97*0.8*3.88%, and bond maturity at 30+0=30. Case #7 has the swaption strike of 1*0.8*3.88% and the bond maturity (-3) +30 = 27.

Column "proxy portfolio – RSM" has the error % of the RSM applied to the proxy portfolio. Column "sensitivity portfolio – RSM" has the error % of the RSM applied to the sensitivity portfolio. "Sensitivity portfolio – adjusted RSM" has the error % of the adjusted RSM applied to the sensitivity portfolio. Column "Is Adjusted RSM better" compares the RSM error with the adjust RSM error of the sensitivity portfolio.

We see that, for the case #7 - #12, sensitivity to the bond maturity, the adjusted RSM is much better than the RSM. For case #2-6, sensitivity to swaption strike, we see that for strike close to the original strike, 0.98 – 1.02, the adjusted RSM performs better. For strike of 0.97, adjusted RSM has larger error than the RSM.

It is intuitive that the performance of the adjusted RSM depends on the similarity between sensitivity portfolios and the proxy portfolio. It performs better for similar portfolios of similar payoffs.

The following Table 3 has the sensitivity result for the proxy portfolio with the swaption strike at K= 1.0*3.88% and bond maturity at 30. We see that in this case, ARSM improved the results for sensitivity portfolio with various maturities, but less so for the sensitivity to the swaption strikes.

Table 3, Performance of Adjusted RSM (M = 30, Strike = 1.0, ATM)

Case #	duration sensitivity	strike sensitivity	proxy portfolio - RSM	sensitivity portfolio - RSM	sensitivity portfolio - adjusted RSM	Is Adjusted RSM better?
13	0	0.97	-3.48%	-0.35%	4.30%	
14	0	0.98	-3.48%	-1.36%	2.98%	
15	0	0.99	-3.48%	-2.40%	1.54%	TRUE
16	0	1	-3.48%	-3.48%	0.00%	TRUE
17	0	1.01	-3.48%	-2.45%	2.99%	
18	0	1.02	-3.48%	-1.58%	5.61%	
19	-3	1	-3.48%	-3.31%	0.82%	TRUE
20	-2	1	-3.48%	-3.37%	0.55%	TRUE
21	-1	1	-3.48%	-3.42%	0.28%	TRUE
22	0	1	-3.48%	-3.48%	0.00%	TRUE
23	1	1	-3.48%	-3.53%	-0.28%	TRUE
24	2	1	-3.48%	-3.58%	-0.57%	TRUE

The following Table 4 has the sensitivity result for the proxy portfolio with the swaption strike at K = 1.2*3.88% and bond maturity at 30. We see that in this case, ARSM did not improve the result. Note that in this case, as the proxy portfolio has a high strike, it is in the money, the curvature of the payoff is not significant. The RSM is working well already in this case.

Table 4, Performance of Adjusted RSM (M = 30, Strike = 1.2)

Case #	duration sensitivity	strike sensitivity	proxy portfolio - RSM	sensitivity portfolio - RSM	sensitivity portfolio - adjusted RSM	Is Adjusted RSM better?
25	0	0.97	0.05%	1.12%	3.32%	
26	0	0.98	0.05%	0.80%	2.36%	
27	0	0.99	0.05%	0.44%	1.27%	
28	0	1	0.05%	0.05%	0.00%	TRUE
29	0	1.01	0.05%	-0.39%	-1.51%	
30	0	1.02	0.05%	-0.85%	-3.12%	
31	-3	1	0.05%	0.04%	0.44%	
32	-2	1	0.05%	0.04%	0.29%	
33	-1	1	0.05%	0.05%	0.15%	
34	0	1	0.05%	0.05%	0.00%	TRUE
35	1	1	0.05%	0.05%	-0.14%	
36	2	1	0.05%	0.06%	-0.29%	

5 Conclusion

In this research, we demonstrated that the adjusted RSM improves the RSM accuracy for payoff functions with significant curvature. As annuity contract and insurance contract in the market has more embedded options, and the regulatory framework moves to a principle based approach. More companies might use the RSM for the reserve calculation. The proposed Adjusted RSM improved RSM for option payoffs.

For the further research, we would define the parameters of the usual life policy or an annuity contract, define the grid based on the key driver of the payoff function of those “proxy” policy / contracts. For example, the moneyness of the guaranteed minimum benefit (GMXB) would be a critical parameter in choosing the percentile points.

Then use the well investigated distribution of the “proxy” policy / contracts, to apply to the company specific products. The authors believe that, ARSM will produce superior results.

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