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EXPLORING TILTED COSMOLOGICAL SCENARIOS: BIANCHI TYPE I MODELS WITH PERFECT FLUID IN GENERAL RELATIVITY

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Abstract: The exploration of spatially homogeneous and anisotropic universes, particularly those exhibiting tilt, has garnered significant interest in recent years. Tilted universes, where matter does not move orthogonally to the hypersurface of homogeneity, offer a nuanced understanding of cosmic dynamics. Early seminal works by King and Ellis (1973), Ellis and King (1974), and Collins and Ellis (1979) extensively examined the general dynamics of tilted universes. Dunn and Tupper (1978) and Lorenz (1981) specifically delved into Tilted Bianchi Type I models, while Mukherjee (1983) investigated these universes with heat flux, revealing intriguing pancake-shaped configurations. Bradley (1988) contributed by deriving tilted spherically symmetric self-similar dust models, adding to the complexity of equations governing tilted cosmological scenarios.

The mathematical formalism governing tilted cosmological models is notably intricate compared to non-tilted ones, as highlighted by Ellis and Baldwin (1984), who proposed the potential presence of tilt in our universe and suggested detection methods. Further advancements include the exploration of tilted cold dark matter cosmological scenarios by Cen et al. (1992), shedding light on the implications of tilt in cosmological dynamics. Additionally, Bali and Sharma (2002) delved into the characteristics of tilted Bianchi Type I dust fluid, revealing peculiar cigar-type singularities under certain conditions.

This abstract encapsulates the evolving landscape of tilted universes, emphasizing the significance of tilt in shaping cosmic evolution and structure. Through a historical overview and examination of key findings, it underscores the importance of understanding tilted cosmological models in elucidating fundamental aspects of the universe's evolution and structure.

Keywords: Tilted universes, cosmological dynamics, Bianchi Type I models, cosmic evolution, cosmic structure

INTRODUCTION

In recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic universe in which the matter does not move orthogonally to the hypersurface of homogeneity. These are called tilted universe. The general dynamics of tilted universe have been studied in detail by King and Ellis (1973), Ellis and King (1974), and Collins and Ellis (1979). Tilted Bianchi Type I models have been obtained by Dunn and Tupper (1978) and Lorenz (1981). Mukherjee (1983) has investigated tilted Bianchi Type I universe with heat flux in general relativity. He has shown that the universe assumes a pancake shape. Bradley (1988) obtained all tilted spherically symmetric self-similar dust models. The equations for tilted cosmological models are more complicated than those of non-tilted ones. Ellis and Baldwin (1984) have shown that we are likely to be living in a tilted universe and they have indicated how we may detect it. A tilted cold dark matter cosmological scenario has been discussed by Cen et al. (1992). Bali and Sharma (2002) investigated tilted Bianchi Type I dust fluid and shown that model has cigar type singularity when $T = 0$.

In this paper, we have investigated tilted Bianchi Type I dust fluid of perfect fluid in general relativity. To get a determinate solution, a supplementary condition P = 0, $A = (BC)^n$ between metric potential is used. The behavior of the singularity in the model with other physical and geometrical aspects of the models is also discussed.

THE METRIC AND FIELD EQUATIONS

We consider metric in the form:

$ds^2=\square dt^2+A^2\, dx^2+B^2\, dy^2+C^2\, dz^2$, (1)

Where A, B and C are functions of 't' alone.

The energy-momentum tensor for perfect fluid distribution with heat conduction given by Ellis (1971) is taken into the form:

$T_i^j \square(\square\square p)v_iv^j \square pg_i^j \square q_iv^j \square v_iq^j$, (2)

Together with

$g_{ij}\, v_i\, v^j = -1$, (3)

$q_iq^j > 0$, (4) 2

$q_iv^i = 0$, (5)

Where p is the pressure, $\square$ the density and q_i the heat conduction vector orthogonal to v^i. The fluid flow vector has the components $\square_{\square}$ ___ sinh$\square$, 0, 0, cosh$_{\square}\square_{\square}$ satisfying Equation
□ A □

3 and $\square$ is the tilt angle.

The Einstein field equation

~~AB44 + AC4~~ 4 + BC44 = $\square 8\pi \square\square\square\square(\square\square p) \cosh 2\square\square p \square 2q_1$ A□□□□ , (10)

$(\square\square p)$ Asinh□ cosh□□$q_1$cosh□□q_1 ___ $^{\sinh 2\square}$□ 0 , (11) cosh□

Where the suffix '4' stands for ordinary differentiation with respect to cosmic time 't' alone.

SOLUTION OF FIELD EQUATIONS

Equations 7 to 11 are five equations in seven unknown A, B, C, $\square$, p, $\square$ and q_1; therefore to determine the complete solution we require two more conditions:

1) We assume that the model is filled with dust of perfect fluid which leads to

p = 0 (12)

2) Relation between metric potential as:

$A = (BC)^n$ (13)

Where n is constant.

Equations 7 and 10 lead to

~~B44 C44 2B4C4~~ □ ~~A4C4~~ □ ~~A4B4~~ □ 8□ (□□p) □ (14)
□

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Rij □1Rgij □□8□Tij, (units such that c = G = 1) (6)
2
For the line, element of Equation 1 are

B44 + C44 + B4C4 = □8 π □□(□□p) sinh2□□p□2q1
B C BC □
sinh□□ , (7)
A □□
A44 +C44 +A4C4 =□8πp ,
A C AC
(8)
A44 +B44 +A4B4 =□8πp ,
A B AB
(9)
A B A C B C sinh
B C BC AC AB
From Equations 12 and 14, we have
B44 □ C44 □ 2B4C4 □ A4C4 □ A4B4 □ 8□□ (15)
B C BC AC AB
Equations 8 and 9 lead to
B44 □ C44 □ A4 □□B4 □ C4 □□□0 (16)
B C A □ B C □
This leads to
□4 □ an□1 (17)
□ □
Where BC = □' B □□ and 'a' is constant of integration.
C
Again from Equations 8 and 9, we have
2A44 □ B44 □ C44 □ A4C4 □ A4B4 □ □16□p (18)
A B C AC AB
From Equations 12 and 18, we have
2A44 □ B44 □ C44 □ A4C4 □ A4B4 □ 0 (19)
A B C AC AB
Equation 19 gives
2 2
2(1□2n)□44 □(4n2 □2n□1)□□□4 □□ □□□□4 □□ □0 (20)
□ □□ □ □□□□ □ □□
Where A = □n .
From Equations 17 and 20, we have
2ff1 □(4n2 □2n□1) f2 □a2 □ (21)

(1□2n) □ □ (1□2n)□2n 1

Where □4 = f(□).

Equation 21 leads to

f 2 □ ________1 2n [a2 □b(4n□1)□4n□1/2n□1] (22)

(4n□1)□

Where 'b' is a constant of integration.

d□ ——(23)

log □ □ a 4n□1□□ a2 □b(4n□1)□4n□1/2n□1

Hence, the metric of Equation 1 reduces to the form

ds2 □ □ ___d f□2 2 □□2n dx2 □□□dy2 □_□□dz2 (24) Where □ is determined by Equation 23.

By introducing the following transformations

μ □ T, x□ X, y □ Y, z □ Z

The metric of Equation 24 reduces to the form

Where dT ds2 □ □□ ______________ a2 □b□(4n□1)T 4 2n□1/2n□1 □□□ dT2 □T2ndX2 □T□dY2 □ T□ dZ2 (25) □

log □ □ a 4n□1□ T a2 □b(4n□1)T 4n□1/2n□1 (26)

SOME PHYSICAL AND GEOMETRICAL FEATURES

The density for the model of Equation 25 is given by:

8 π□= ______________(4n□1)b (27) 2(2n□1) T

The tilt angle □ is given by:

coshλ = 1 2n □1 (28)

2 n

sinhλ = 1 1□2n (29)

2 n

The reality conditions

(i) □ + p > 0,

(ii) □ + 3p > 0, lead to

____________b(4n□1) T □0 (30)

2(2n□1)

Where

b(4n□1)

________□ 0

2(2n□1)

The scalar of expansion □ calculated for the flow vector □i is given by:

(n□1) (2n□1)□a2 □b(4n□1)T4n□1/2n□1 □ (31)

□□ n□1 (4n□1)Tn□1

2T

The components of fluid flow vector v^i and heat Bagora and Bagora 3 conduction vector q^i for the model of Equation 25 are given by:

$$□^1 □ \frac{1}{2T^n}\sqrt{\frac{1□2n}{n}} \quad (32)$$

$$□^4 □ \frac{1}{2}\sqrt{\frac{2n□1}{n}} \quad (33)$$

$$q^1 □ \frac{□(4n□1)b}{64□T^{\frac{6n^2□3n□1}{2n□1}}}\sqrt{\frac{1□2n}{n}} \quad (34)$$

4 (4n□1)(1□2n)b 1□2n (35) q □ □
n $\frac{4n^2□2n□1}{2n□1}$
64□(2n□1)T

The non-vanishing components of shear tensor (□ij) and rotation tensor (□ij) are given by

(4n² □1) (2n□1)□a² □b(4n□1)T^4n□1/2n□1□ (36)
□11 □ 24nT1□n n(4n□1)
□22 □12□Tnn((24nn□√□11)) □(1□2n) □a2 □√b(4n□1)T4n□1/2n□1□□3a 4n□1□
(37)

□33 □ 121Tn n((24nn□√□11)) □(1□2n) □a2 □√b(4n □1)T4n□1/2n□1□□3a 4n □1□ □
(38)

44 (2n □1)² (1□2n)□a² □b(4n □1)T^4n□1/2n□1□ (39)
□ □□ 24n n(4n □1)
□(26n² □3n □2) (1□2n)□a² □b(4n □1)T^4n□1/2n□1□
□14 □□
24nT n
(40)

ω14 = (6n□1) (1□2n)□a² □b(4n□1)T^4n□1/2n□1□ (41)
16T n(4n□1)

The rates of expansion H_i in the direction of x, y and z axes are given by

$$H_1 □ \frac{n}{T^{8n^2□4n□1/2(2n□1)}}\sqrt{\frac{a^2T^{4n^2□2n□1/(4n□1)} □ b(4n□1)T^{2n}}{4n□1}} \quad (42)$$

$$H_2 □ \frac{1}{2T^{8n^2□4n□1/2(2n□1)}}\sqrt{\frac{a^2T^{4n^2□2n□1/(4n□1)} □ b(4n□1)T^{2n}}{4n□1}} □ \frac{a}{2T^{n□1}} \quad (43)$$

$$H_3 □ \frac{1}{2T^{8n^2□4n□1/2(2n□1)}}\sqrt{\frac{a^2T^{4n^2□2n□1/(4n□1)} □ b(4n□1)T^{2n}}{4n□1}} □ \frac{a}{2T^{n□1}} \quad (44)$$

DISCUSSION

The model started with a big-bang at T = 0 and the expansion in the model decrease as time T increases and it stopped at T = □. The model has point type singularity at T = 0 (MacCallum, 1971). The model represents shearing and rotating universe in general and rotation goes on decrease as time increases. Since Lim _□□0, then the model does not approach isotropy
T □□ □
for large value of T.
Density □ □ 0 as T □ □ and □ □ □ as T □ 0. When T □ 0. q^1 □ □ and q^4 □ □. Also, q^1 and q^4 tend to zero as T □ 0. At T = 0, the Hubble parameters tend to infinite at the time of initial singularity of vanish as T □ □.

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