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INTERVAL ANALYSIS FOR THE IDENTIFICATION OF NONLINEAR MODELS IN STATIC SYSTEMS

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ABSTRACT: The article addresses a significant scientific problem – the development of identification methods for interval nonlinear models of static characteristics of complex objects with acceptable computational complexity. It examines the challenges associated with identifying the parameters and structure of nonlinear models of static characteristics. The proposed solutions reduce the complexity of the modelling process while ensuring the derivation of adequate models with guaranteed accuracy, determined by experimental results in the form of interval values of the modelled characteristics. A parameter identification approach for interval nonlinear models is presented, which reformulates the problem as minimizing the quadratic deviation between the modelled characteristics of a static object and the experimental intervals. Although this approach expands the optimization parameter space by introducing additional coefficients into the objective function to ensure consistency between experimental data and calculations, it also enables the development of efficient optimization procedures. For structural identification, a method based on analyzing the gradient of the objective function of the optimization problem is proposed, allowing for the directed selection of structural elements during the synthesis of an interval nonlinear model. A novel structural identification method for nonlinear interval models and an algorithm for its implementation have been developed. Experimental examples confirm the high convergence and efficiency of the proposed approach. The proposed methods for nonlinear model identification based on interval data analysis will contribute to the advancement of applied research in national security, environmental monitoring, medicine, and other fields where mathematical models serve as the foundation for decision-making.

KEYWORDS interval model, structural identification, parametrical identification, interval data, optimization problem, objective function, gradient.

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INTRODUCTION requires additional processing to develop mathematical and mathematical modelling is one of the most effective tools algorithmic support for modelling software tools [23, 24, and 35]. for conducting research and solving a wide range of The authors of the interval approach [12, 34] state that it has Applied problems where complex decisions need to be made, particularly with the help of intelligent decision support systems and expert systems [9, 17, 44, 45]. Proximity to the user and convenience in the application of mathematical modelling methods are implemented through the use of various software environments, which mainly integrate the spectrum of mathematical methods into a single concept of building tools that are understandable for practical users [3, 5, 8, 10, and 41]. Examples of such environments are MATLAB, Mathematical, or Python programming libraries such as NumPy and SciPy. At the same time, the need to use specialized methods developed by some researchers, such as interval data analysis methods, some advantages over the stochastic (probabilistic) approach [14, 36]. Among them is the absence of a requirement to study the statistical characteristics of the modelling object [2, 17]. As it is known, this reduces the number of experiments (selection and accumulation of data). Therefore, the interval approach is more useful for studying the characteristics of the complex object in the conditions of a limited data sample [33]. The main concepts of this approach should be considered in detail to develop decision support systems oriented to data, models, and knowledge. When researching objects and processes across different fields, it becomes a task to establish cause-and-effect relationships and dependencies for the complex objects' static Characteristics and factors representing the external environment influence. In this case, these objects are often treated as static systems, disregarding transient processes, and constructed the "black box" type mathematical models [20]. Constructing models of static systems includes problems of parametric and structural identification. Many methods have been developed for the parametrical identification of interval models for dynamic and static systems [22, 26, 27, and 43]. It's worth noting that the interval models' computational optimization parametric identification problem is an NPcomplex problem. The peculiarity in the parametric and structural identification of the static systems interval nonlinear models lies in the necessity of finding the objective function global minimum during optimization while navigating through numerous local minima. As a result, metaheuristic methods employing stochastic search techniques are predominantly used to address this challenge [6, 7, 11, 12, 15]. Among these methods, those grounded in swarm intelligence are currently considered the most computationally efficient. Specialized software tools tailored for these methods have been developed and are extensively utilized by researchers [13, 19, 21]. Additionally, established software solutions provided within standard application packages, such as the Global Optimization Toolbox of MATLAB, are widely utilized in this regard [28, 31, 32, 39, 40]. Analysis of methods for nonlinear interval models' structural identification has revealed that the primary issue contributing to the optimization stochastic and combinatorial nature is selecting structural elements from the set of all possible elements. The predominant approach involves optimizing parameters for each model structure formed through combination, selection, or mutation [21, 29, 37]. The objective of the research is developing identification procedures for both parameters and structure of interval nonlinear models, aiming for an acceptable computational complexity in solving the structural identification problem. This entails developing a method for selecting structural elements that can significantly reduce the parametric identification procedures number during the model's structure synthesis.

MATERIAL AND METHODS

A. STATEMENT OF THE TASK

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As a rule, static characteristics of the complex objects are described as functional dependencies for output values representing object characteristics from input values representing influencing factors in the system. These dependencies are written in the algebraic equations form [28]:

$y(X) = f\ \vec{\beta}, X + f\ \vec{\beta}, X + \cdots + f\ \vec{\beta}, X$, (1) where $y(X)$ is a simulated value of the object static characteristic; $\vec{\beta}$ is a nonlinear parameters vector of the model, that needs to be estimated using experimental data; $\lambda =$ $f\ \vec{\beta}, X\ ,\ f\ \vec{\beta}, X\ ,\ \ldots\ ,\ f\ \vec{\beta}, X$ is a basis nonlinear functions set relative to the input variables X and the model parameters vector, nonlinear functions can be used depending on the objects specifics being studied [39]:

- Indicators: $f\ \vec{\beta}, X \quad = \beta \cdot X$;
- Gaussian models: $f\ \vec{\beta}, X \quad = \beta \cdot e$;
- Trigonometric Fourier series: $f\ \vec{\beta}, X = \beta \cdot cos(\beta \cdot X) + \beta \cdot sin(\beta \cdot X)$, etc., where m is the given number of basic functions of the model (structural elements). The experiment results that are used for the identifying of nonlinear (in general) models (1) are obtained in the form:

$\vec{X} = (x \cdots x \quad) - \quad [y\ ;\ y\ ],\ i = 1, N,$ (2)

where $[y\ ;\ y\ ]$ is experimentally obtained static characteristics values of the nonlinear object, which are presented as the upper and lower limits in the ith measuring, $i = 1, N, \vec{X}$ is the influencing factors value (input variables) on the object characteristic in the i-th measuring; N is the total number of measurements of the experiment. Let's give basis functions set λ for a model, and parameters vector $\vec{\beta}$ interval estimates are obtained. Then the dependence that describes the interval model of the static characteristics of the nonlinear objects from the input values will have such form of the nonlinear algebraic expression:

$\hat{y}\ \vec{X} \quad = f \quad \vec{\beta} \quad , \vec{X} \quad + \cdots + f \quad \vec{\beta} \quad , \vec{X} \quad , i = 1, N,$ (3)

where $\hat{y}\ \vec{X} = \hat{y}\ \vec{X}\ ;\ \hat{y}\ \vec{X}$ are the modelled characteristic interval estimates that are calculated based on input values X, $\vec{\beta} = \vec{\beta}, \vec{\beta}, \ldots, \vec{\beta}$ is a model parameters interval estimates vector.

Considering the condition belonging to the interval estimates $\hat{y}\ \vec{X}$ to the interval values of object characteristics that are obtained experimentally,

$\hat{y} \quad \vec{X} \quad ;\ \hat{y} \quad \vec{X} \quad \subset [y\ ;\ y\ ],\ i = 1, N,$ (4)

a mathematical problem calculation interval estimates of the model parameters vector $\vec{\beta}$ we get [20]:

$\{y \quad \leq f \quad \vec{\beta} \quad , \vec{X} \quad + \cdots + f \quad \vec{\beta} \quad , \vec{X} \quad \leq y\ ,\ i = 1, N.$ (5)

It's an interval system of nonlinear algebraic equations (ISNAE) concerning model parameters vector $\vec{\beta}$ interval estimates. There's solutions set ISNAE Ω determining interval estimates $\vec{\beta}$ of the model parameters vector. In practice, only point-based parameter estimates are calculated, which is connected with solving ISNAE high computational complexity. In this case, an optimization problem to pointbased estimate the parameters in the form is solving [24]:

$\delta\ \vec{\beta} \quad \xrightarrow{,} \quad min$ (6)

$\alpha \in [0,1],\ i = 1, N.$ (7)

where λ is a set of elements that are used to synthesize the structure of the interval model and contains all possible given structural elements; s is a number of all possible elements of the structure; αi are the linear combination coefficients that are

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used for calculating a point that belongs to the experimental data limits.

The objective function $\delta\ \vec{\beta}$ in expression (6) is formed based on considering the constraints set by the ISNAE (5) [22]. The objective function is minimizing the point-based model quadratic error:

$\hat{y}\ \vec{X} = f\ \ \vec{\beta}, \vec{X} + \cdots + f\ \ \vec{\beta}, \vec{X}\ , i = 1, N,$

and looks like this (8)

$\delta\ \vec{\beta} = \sum\ \ \hat{y}(\vec{X}) - P([y\ ; y\ ], \alpha\ )$, (9)

where

$P([y\ ; y\ ], \alpha\ ) = \alpha \cdot y\ + (1 - \alpha\ ) \cdot y\ , i = 1, N.$ (10)

The suggested article [22] method for parametrically

Identifying static systems interval nonlinear models involve simplifying the identification task. Strategy based on expanding the parameter space of nonlinear models by incorporating extra coefficients into the objective function. Consequently, we encounter a nonlinear optimization task for which gradient methods with polynomial complexity are used. However, when the objective function becomes complicated, or discrete through the nonlinear parameters, there used alternative optimization techniques, such as stochastic, evolutionary, and metaheuristic approaches [1, 14, 19, 21].

Simultaneously, the structural identification task involves determining both model's structure which represented by the structural elements set $\lambda \in \lambda$, and parameters $\vec{\beta}$. Structural identification entails transforming the set λ to a vector form, $\vec{\lambda} = l^{()}, l^{()}, l^{()}$ by binary, decimal coding or hashing.

Let's write the structural identification problem based on

(6) In the following form:

$\delta\ \lambda, \vec{\beta} \longrightarrow min,$ (11)

$\lambda\ \ \in \lambda$, (12)

$\alpha \in [0,1], i = 1, N.$ (13)

Condition (4) is used as a stop-criterion during the optimization for the case of the point-based model [27]:

$\hat{y}\ \vec{X} \subset [y\ ; y\ ], i = 1, N.$ (14)

However, the challenge lies in the discrete nature of this function, as optimization relies on discrete values of vector, these discrete values originate from a specific coding scheme applied to elements within set λ . Consequently, resolving this problem involves addressing multiple instances of the parametric identification problem (6) while selectively choosing sets λ from elements within set λ . Now, selecting or developing computational methods and means for identifying static systems interval models based on optimization problems (11)-(13) is actual. Simultaneously, the quality criteria of such methods are independence from input data, low computational complexity, and finding a global minimum.

B. THE METHOD OF IDENTIFYING INTERVAL NONLINEAR MODELS STRUCTURE USING THE OBJECTIVE FUNCTION GRADIENT ANALYSIS

The smoothness of function (11) makes it possible to study it using gradient methods [4, 5, 25]. Consider a candidate model characterized by a structure defined by vector λ and a parameter vector $\vec{\beta}$. To evaluate the candidate model structure quality, we suggest utilizing the objective function $\delta\ \vec{\lambda}, \vec{\beta}$ anti-gradient at the point $\vec{\beta}$. The magnitude

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of the anti-gradient vector $-\nabla$ calculated based on the vector of the parameters $\vec{\beta}$ determines the direction for the objective function minimizing $\delta\ \vec{\lambda}, \vec{\beta}$ for the candidate model structure based on vector $\vec{\lambda}$. Thus, the anti-gradient vector for a structure λ we denote:

$-\nabla\delta\ \vec{\lambda}\ , \vec{\beta}\ =\ \_\_\_\_\_\_\ \_\_\_\_\_\_\vec{\ }\ ,\vec{\ }\ , . . . ,\ \vec{\ }\ ,\vec{\ }\ .$ (15)

Utilizing the property of multidimensional functions derivative concerning its variables, we can formulate an optimality condition of the structural elements set λ for a candidate model, regarding the problem (11-13). Accordingly, the minimization of the anti-gradient vector norm

$-\nabla\delta\ \vec{\lambda}\ , \vec{\beta}$ at the point $\vec{\beta}$ for the objective function (12) on the whole set λ testifies to the optimality of the setλ

Following the condition, guidelines for selecting structural elements during the structure synthesis based on the objective function gradient within the implementation of the structural identification method are proposed.

Principle 1. The optimality of the structural elements set λ of the candidate model from the point view of the problem (1113) for a fixed parameters number determines the norm minimum of the objective function (11) anti-gradient vector

$-\nabla\delta\ \vec{\lambda}, \vec{\beta}$ at the point $\vec{\beta}$.

If the rule is executing, we proceed with one of two courses of action: if condition (5) is met, the problem (11-13) solution has been discovered; otherwise, we increment the model dimension accordingly $m + 1$.

Principle 2. The best candidate for selection among all set elements λ is the element that provides an opportunity to minimize the objective function (11). The quality criterion will be the value of the derivative, which is determined based on the model, which includes the selected structural element. Accordingly, the largest norm value of the anti-gradient for the formed set λ will indicate the possibility of the best objective function minimization. The objective function derived value

$\_\_\ \delta\ \beta\ , f\ \vec{\beta}\ , X$ based on the , $\vec{\ }, \vec{\ }, \vec{\ }$

model, for which the objective function minimum is determined considering the new structural elements $f\ \vec{\beta}\ , X \in \lambda \wedge f\ \vec{\beta}\ , X \notin \lambda$, $w = 1, s$, is proposed to use for the quantitative evaluation of the new structural elements.

In Figure 1 an example is illustrated, which demonstrates that this approach ensures the rejection of structural elements that worsen the quality of the model, since for them there is no value of the derivative at a given point and the possibility of choosing the best structural element for the current model. At the same time, the value of the derivative will indicate the ability of the structural element to ensure the objective function minimization. Accordingly, it is necessary to choose a structural element with the maximum value of the norm of the derivative. In Figure 1(a) shows the graphs of the objective

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function based on three models with the structures .

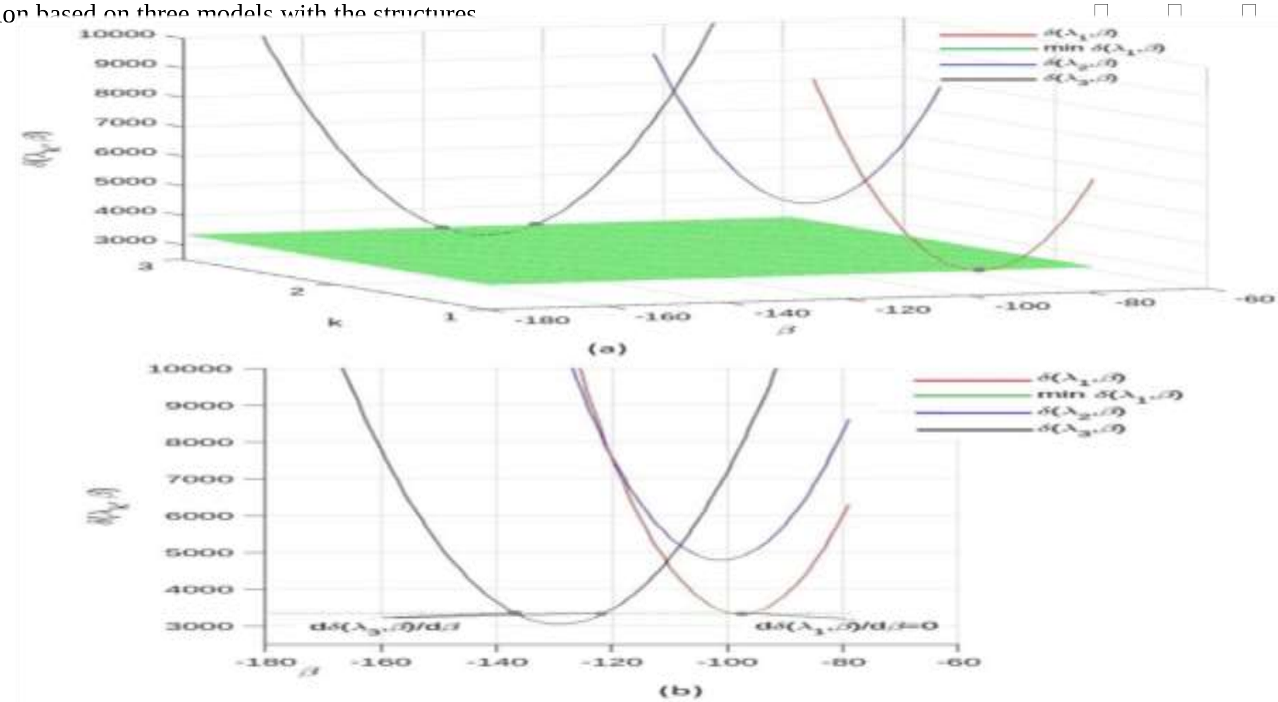

Figure 1. Illustration of the selection procedure of a structural
Element based on the evaluation of the objective function gradient

At the same time, the objective function $\delta \vec{\lambda}, \vec{\beta}$ global minimum is determined based on the model with the structure λ. The value of the minimum is used to find the derivative of the objective function in order to select new structural elements that will ensure its minimization. As we can see in Figure 1(b), the structural element that forms the structure λ does not ensure the minimization of the objective function and is not taken into account to form the current structure. Accordingly, the structural element forming the structure λ is a candidate for selection.

The algorithm is proposed for nonlinear interval model structural identification that minimizes the objective function within the parameter space, employing discrete values-directed selection for the model's structure. The algorithm pseudocode for nonlinear interval model structural identification is given in Figure 2.

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Algorithm 1: Interval Nonlinear Model Structural Identification
Initialization:
set: possible structural elements set, λ_s;
calculate the size structural elements set s = length(λ_s);
iteration counter k = 1;
the initial structure
        λ_k = {f_1^k(β_1^k, X), ..., f_{m_k}^k(β_{m_k}^k, X)};
value of criteria of stopping Stop_Criterion = FALSE;
while Stop_Criterion == FALSE do
    calculate the size model structure set m_k = length(λ_k);
    estimate the vector of model parameters β^k;
    if model_adequate() then
        Stop_Criterion = TRUE;
    else
        for v ← 1 to m_k do
            for w ← 1 to s do
                calculate the objective function derivative by the v-th
                 parameter for the w-th structural element
                        d/dβ_v δ(β_v^k, f_w^s(β^k, X));
            end
        end
        search the anti-gradient vector based on the k-th structure
         −∇δ(λ_k, β^k);
        for v ← 1 to m_k do
            Replace in structure λ_k according to rule 2 element f_v^k(β^k, X)
             on f_w^s(β, X), for which ||d/dβ_v δ(β_v^k, f_w^s(β^k, X))|| = max;
        end
        estimate the vector of model parameters β^k;
        if model_adequate() then
            Stop_Criterion = TRUE;
        else
            k = k + 1;
            Add to structure λ_k a new element f_w^s(β^k, X);
        end
    end
end
Return the set λ_k and vector β^k.
```

Figure 2. The algorithm pseudocode for nonlinear interval model structural identification

To calculate the anti-gradient, we can use a numerical method or a partial derivatives analytical expression for a given structural elements at a point $\vec{\beta}$. d

$\_\_\_ \; \delta \quad \beta \; , f \quad \beta^{\rightarrow} , X \; =$

$d\beta$

d

$= \_\_\_ \quad y \qquad X^{\rightarrow} \qquad - P([y \; ; y \;], \alpha \;) + \beta \quad \cdot f \qquad \beta^{\rightarrow} , X^{\rightarrow} =$

$d\beta$

$= 2 \cdot \quad y \qquad X^{\rightarrow} \qquad - P([y \; ; y \;], \alpha \;) + \beta \quad \cdot f \qquad \beta^{\rightarrow} , X^{\rightarrow} \; \times$

$\times \_\_ \sum \beta \qquad \cdot f \qquad \beta^{\rightarrow} , X^{\rightarrow}$ (16)

where

$y \qquad X^{\rightarrow} \qquad = \qquad \beta \qquad \cdot f \qquad \beta^{\rightarrow} , X^{\rightarrow} \; , q \neq v \; ;$

β is parameter of the model according to which the differentiation of the objective function is carried out $\beta \in \; \beta^{\rightarrow}$, f $\beta^{\rightarrow}$, X is a structural element candidate for selecting,

$f \qquad \beta^{\rightarrow} , X^{\rightarrow} \in \lambda \wedge f \qquad \beta^{\rightarrow} , X^{\rightarrow} \notin \lambda$.

III. RESULTS AND DISCUSSION

We will conduct experiments in modelling objects considered static systems to test the proposed structural and parametric identification approach. Let's consider an example of using the structural identification method to simulate the daily electricity generation by a mini hydroelectric power plant (MHPP). We will also consider the case of applying the method of structural identification with a known basis, determined by the feature of the modeled object, using the example of modelling the distribution of the ground level of pollution from a point source, which, as is known, is described by Gaussian models [25, 38, 49].

A. MODELING OF THE GENERATED POWER BY MHPP The task of restoring existing and creating new mini hydroelectric plants is urgent, considering the potential of hydro resources in Ukraine. At the same time,

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developing mathematical models of hydropower plant characteristics is expedient to research and ensure the maximum efficiency of using hydropower resources [46, 47]. As an example of such studies, the “Topolky” MHPP, built on the Strypa River in the Ternopil region, was chosen. The specified MHPP has two turbines connected to generators with a 70 kW and 90 kW capacity. The operation of generators in the system requires a constant assessment of the state of the characteristics of hydro resources and forecasting of the possible generated electricity to use the plant’s equipment sparingly. In particular, it is necessary to forecast each time weather conditions change and seasonal fluctuations of available hydro resources to make decisions about the feasibility of using two turbines at the same time or whether using one of the two turbines is appropriate. In this case, we can take off one of the turbines for repair. Thus, there is a need to develop and use a model that relates the amount of potentially generated electricity depending on the characteristics of hydrotechnical equipment and available hydro resources. We will use the developed methods of structural identification of nonlinear models to identify this mathematical model. Experimental data were obtained as a result of the research of this MHPP. The amount of electricity produced per day is presented in an interval form due to errors in estimating this value by technical means. The reactive power of the turbine and the water pressure, which is determined by the biefs difference, and the height of the water level at the hydro post, are taken as the factors that shape the working conditions of the MHPP and, accordingly, affect the amount of generated power. The characteristic that is simulated is the generated power. Measurements were made daily. Accordingly, the task is to establish cause-and-effect relationships between the observed characteristics of the MHPP and influencing factors. The set of all potential structural elements for the model of the dependence of the generated power on the operating conditions of the MHPP is formed:

$\beta \cdot x , \beta \cdot x , \beta \cdot x , \beta \cdot x , \beta \cdot x , \lambda = \beta \cdot x , \beta \cdot x \cdot x , \beta \cdot x \cdot x , \beta \cdot x \cdot x ,$ (17)

$\beta \cdot x \quad \cdot x \qquad , \beta \cdot x \ \cdot x \qquad , \beta \cdot x \ \cdot x \qquad .$

Using the method of structural identification given above, the following structure of the interval model of daily electricity generation was obtained:

$y(\lambda , X) = \beta \cdot x \cdot x \quad + \beta \cdot x ,$ (18)

and the results of parametric identification for the obtained model: $\vec{\beta}$ = (88.619, 0.4256, 2.5533, 0.4914), $\vec{\alpha}$ = (0.1985,
0.273, 0.1713, 0.0865, 0.6162, 0.9991, 0.8814, 0.397, 0.7368,
0.5845, 0.5823, 0.4998, 0.4427, 0.5582, 0.2692, 0.1964, 0.0002, 0.9202, 0.675, 0.3728, 0.4643, 0.7596, 0.9164, 0.9667,
0.2294, 0.9564, 0.7288, 0.3038, 0.9127, 0.1438).

According to this, a point model was built based on interval data and structure (18) in the following form:

$y(X) = 88.619 \cdot x \cdot x^{\cdot} \qquad + 2.553 \cdot x^{\cdot} \quad .$ (19)

Figure 3(a) shows a graphical representation of the satisfied condition (15) for the obtained model, that is, the inclusion of values predicted based on model (19) in the experimental

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corridor obtained based on measurements.

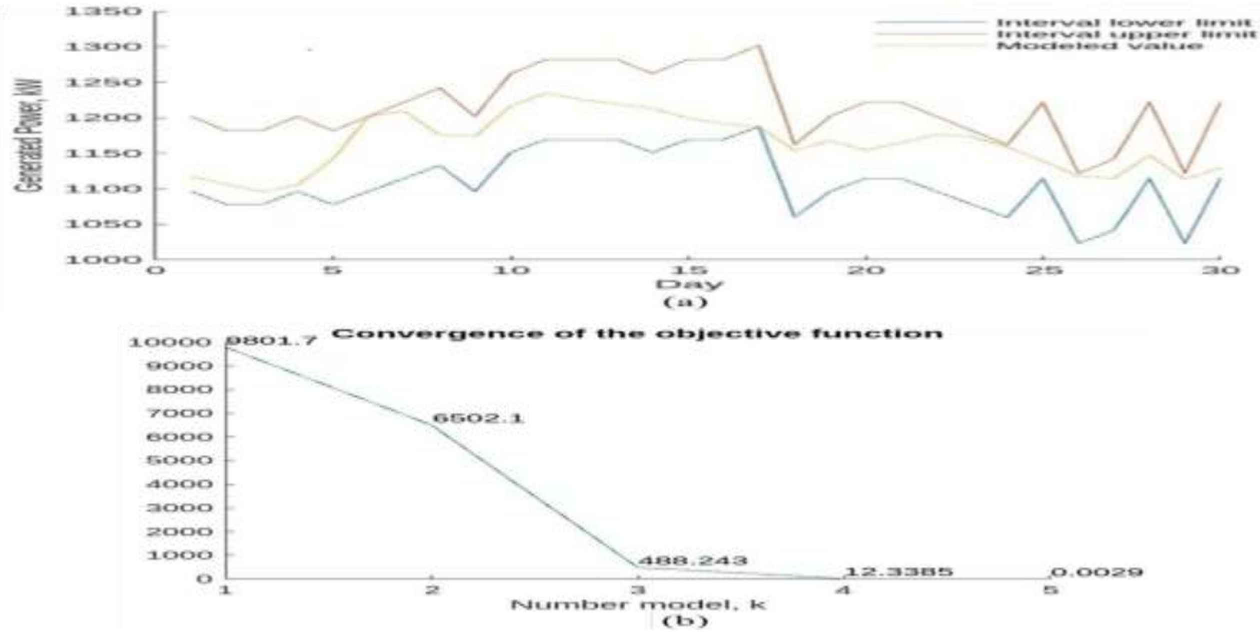

Figure 3. Results: (a) graphic representation of the satisfied Condition (15) for the obtained model, (b) convergence of the structural identification algorithm for model (19)

The effectiveness of the proposed approach and the algorithm for its implementation demonstrates the convergence of this algorithm during the structural identification of the model (19) in Figure 3(b). As we can see, for the input set λ of 12 structural elements, the number of evaluated models was five, which indicates the effectiveness of this method despite the additional computational costs related to analyzing the derived objective function.

B. MODELLING OF THE GROUND LEVEL OF HARMFUL EMISSIONS SPREAD FROM THE POLLUTION SOURCE

Modelling the ground level of the spread of harmful emissions from the source of pollution is an important tool for assessing the impact of pollution on the environment and human health.

The main modelling steps include [48, 49]:

− Determination of the source of pollution;
− Determination of pollution characteristics;
− considering meteorological and other conditions necessary to determine the path of the spread of pollution.

The Figure 4(a) shows the principle of formation of the background level of pollution by harmful SO emissions based on the spread of the plume of pollution from the source of pollution, which is the pipe of an industrial facility, under dry weather conditions. Dry weather ensures the deposition of harmful substances along the spreading plume. Since high humidity causes a high level of SO fluctuation, for other weather conditions the formation of the ground level will have a different character [30]. An industrial object and experimental measurement data from the CECR research project were considered the research object [16]. Experimental data in Table 1 don't include meteorological data. The modelling is based on the conditions that the spreading is carried out uniformly around the pollution source.

Table 1. Experimental Data Measurement SO

Measurement points	Distance from source, m	Interval lower limit, units/m3	Interval upper limit, units/m3

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1	100	2.5803	3.1537
2	200	247.86	302.94
3	300	528.93	646.47
4	400	569.79	696.41
5	500	510.57	624.03
6	600	432.63	528.77
7	700	361.62	441.98
8	800	302.94	370.26
9	900	255.96	312.84
10	1000	218.52	267.08
11	1100	188.55	230.45
12	1200	164.34	200.86
13	1300	144.72	176.88
14	1400	128.52	157.08
15	1500	115.02	140.58
16	1600	103.68	126.72
17	1700	94.05	114.95
18	1800	85.86	104.94
19	1900	78.804	96.316
20	2000	72.684	88.836

In this case, the specificity of the problem determines the use of the model structure in the form of Gaussian models, since they are classically used for this type of problem. Accordingly, the task of structural identification is reduced to determining the model type and the number of structural elements within a given type. The formed set of all potential structural elements for the model will look like this:

$\lambda = e, k = 1, \ldots s.$ (20)

In the course of building up the structure and parametric identification, we obtained a structure that satisfied the stopping criterion (14):

$y(X) = \beta \cdot e + \beta \cdot e +$

$+\beta \cdot e$, (21)

and the results of parametric identification for the obtained model: $\vec{\beta}$ = (419.8273, 57.4029, 692.11, $5.75 \cdot 10^{16}$, -70379.1,
12357.3, -874.7416, 66.9348, 176.016), $\vec{\alpha}$ = (0.7837, 0.9396,
0.4033, 0.4695, 0.5511, 0.669, 0.7674, 0.8013, 0.7657, 0.6824, 0.5777, 0.4751, 0.3848, 0.3165, 0.2645, 0.2203, 0.1812, 0.1323,
0.0737, 0.0014).

A graphical representation of the fulfilment of condition (11) for the obtained model, that is, the inclusion of values predicted based on model (21) in the experimental corridor obtained based on measurements is shown in Figure 4(b).

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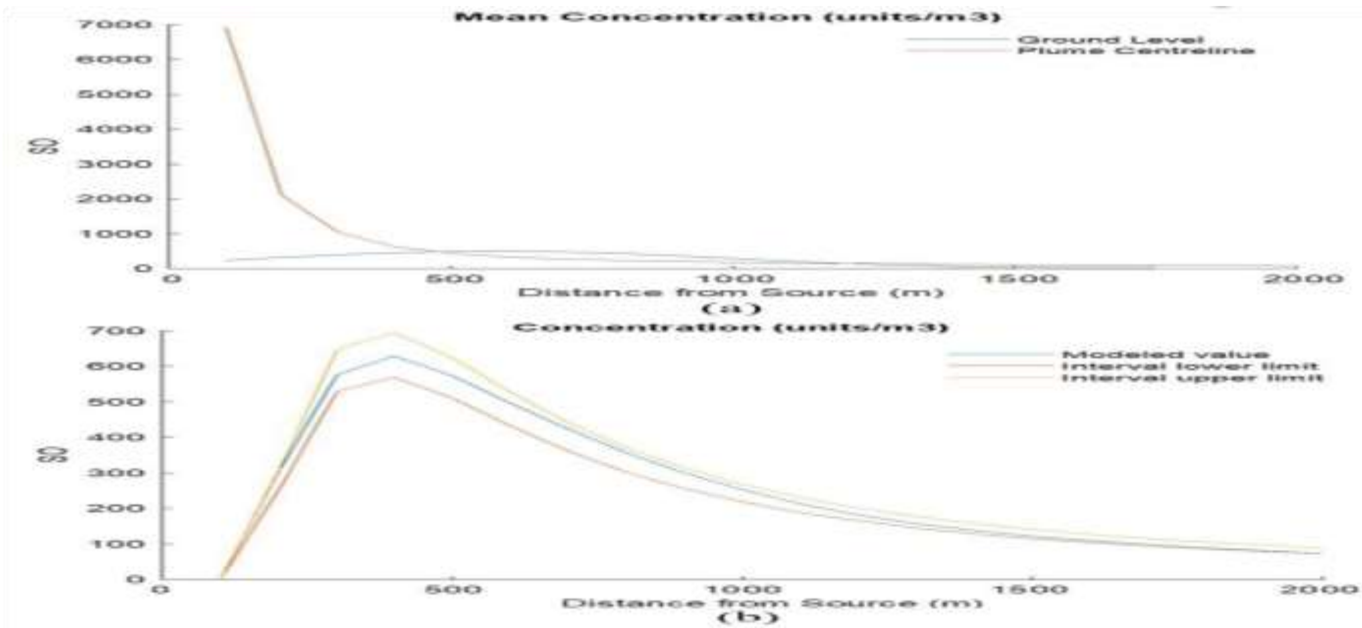

Figure 4. Results of modelling of the ground level of harmful emissions spread

Typically, models of this type are used to construct the distribution of concentrations of harmful emissions [50]. For a point source of pollution, let's plot the distribution of concentrations around the point of the pollution source according to the radius of the experimental measurements in Figure 5(a). For ease of use, let's build a 2D projection (Figure 5(b)) of the ground level concentration distribution around the pollution source. Accordingly, the obtained distribution of concentrations of the ground level of pollution around an industrial object can serve as a tool to support decisions on ensuring environmental safety at the location of such objects.

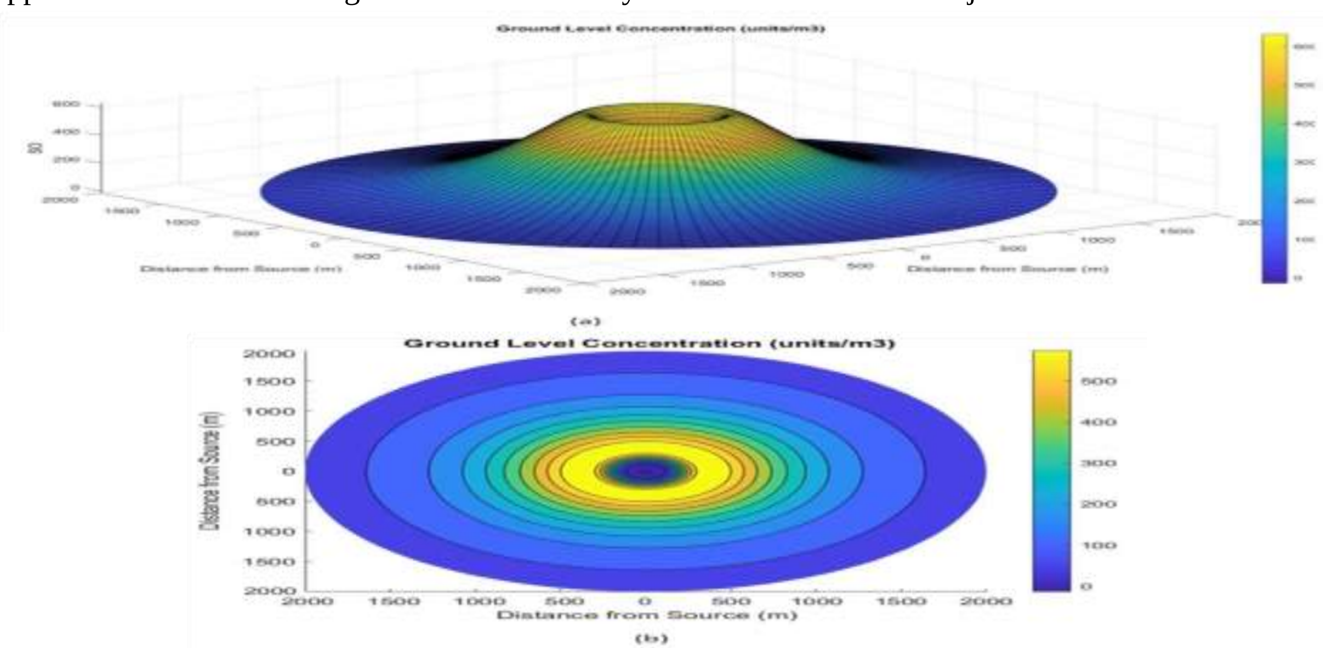

Figure 5. Modelled concentration of the ground level around

The examples presented confirm the effectiveness of the proposed methods of parametric and structural identification. However, it should be noted that their use is limited by the nondifferentiability of the objective

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function of the optimization problem (6)-(7). To apply the proposed methods, the selection of basic functions must be carried out in such a way as to ensure the differentiability of the specified objective function.

IV. CONCLUSIONS

This study examines approaches to the mathematical modeling of static objects. It is established that experimental data are primarily obtained from measuring instruments, which inherently introduce certain measurement errors. Therefore, it is essential to account for uncertainty in experimental data by employing interval data analysis methods. The study addresses the challenges of structural and parametric identification of static object models which can be formulated as optimization problems with nonlinear objective functions. A parametric identification approach for interval nonlinear models is proposed, which reformulates the problem as minimizing the mean squared deviation between the modeled characteristic values of the static object and the corresponding experimental intervals. This approach expands the parameter space of nonlinear models by introducing additional coefficients into the objective function, ensuring consistency between experimental data and model-based calculations. However, it also enables the development of efficient optimization procedures for the realization of methods of parametrical identification of these models. Furthermore, a structural identification method is presented, based on analyzing the gradient of the objective function in the optimization problem to guide the selection of structural elements during the synthesis of an interval nonlinear model, along with an algorithm for its implementation. Fundamental guidelines for selecting structural elements are formulated. This solution reduces the complexity of the modeling process while ensuring the derivation of adequate models with guaranteed accuracy. Experimental examples confirm the high convergence and efficiency of the proposed parametrical and structural identification methods.

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